

# A cosmological bound on $e^+e^-$ mass difference

A.D. Dolgov<sup>a,b,c,d</sup>, V.A. Novikov<sup>a,b,e,f</sup>

<sup>a</sup> Novosibirsk State University, Novosibirsk, 630090, Russia

<sup>b</sup> Institute of Theoretical and Experimental Physics, Moscow, 113259,  
Russia

<sup>c</sup>Dipartimento di Fisica, Università degli Studi di Ferrara, I-44100 Ferrara,  
Italy

<sup>d</sup>Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100 Ferrara,  
Italy

<sup>e</sup> National Research Nuclear University MEPhI

<sup>f</sup> Moscow Institute of Physics and Technology, 141700, Dolgoprudny,  
Moscow Region, Russia

## Abstract

We demonstrate that CPT-violation due to  $e^+e^-$  mass difference generates a non-zero photon mass. As a result the cosmological bounds on the photon mass lead to the bounds on  $e^+e^-$  mass difference which are at least by 10 orders of magnitude stronger than the direct experimental bound.

There is widely spread habit to parametrize  $CPT$  violation by attributing different masses to particle and antiparticle (see PDG [1]). This tradition is traced to the theory of  $(K - \bar{K})$ -meson oscillation. For a given momenta  $\mathbf{q}$  the theory of oscillation is equivalent to a non-hermitian Quantum Mechanics (QM) with two degrees of freedom. Diagonal elements of  $2 \times 2$  Hamiltonian matrix represent masses for particle and antiparticle. Their inequality breaks  $CPT$ -symmetry. Such strategy has no explicit loopholes and is still used for parametrization of  $CPT$ -symmetry violation in  $D$  and  $B$  meson oscillations.

Quantum Field Theory (QFT) deals with an infinite sum over all momenta. It is important that the set of plane waves with all possible momenta for particle and antiparticle form a complete set of orthogonal modes and an arbitrary free field operator can be decomposed over this set.

A naive generalization of  $CPT$ -conserving QFT to  $CPT$ -violating QFT was made by Barenboim et al. (2001) [2]. They represented a complex scalar field as an infinite sum over modes and attributed different masses for particle and antiparticle

$$\phi(x) = \sum_{\mathbf{q}} \left\{ a(\mathbf{q}) \frac{1}{\sqrt{2E}} e^{-i(Et - \mathbf{q}\mathbf{x})} + b^+(\mathbf{q}) \frac{1}{\sqrt{2\tilde{E}}} e^{i(\tilde{E}t - \mathbf{q}\mathbf{x})} \right\} , \quad (1)$$

where  $(m, E)$  and  $(\tilde{m}, \tilde{E})$  are masses and energies for particle and antiparticle with momentum  $\mathbf{q}$  respectively. Here the pairs  $(a^+(\mathbf{q}), a(\mathbf{q}))$  and  $(b^+(\mathbf{q}), b(\mathbf{q}))$  are creation and annihilation operators for particles and antiparticles respectively. They obey the standard Bose commutator relations. In this formalism one can calculate the Wightman functions:

$$< \phi(x), \phi(y)^+ > = D^+(x - y; m) , \quad (2)$$

$$< \phi(x)^+, \phi(y) > = D^-(x - y; \tilde{m}) . \quad (3)$$

They are given by the standard Lorentz-invariant Pauli-Jordan functions but with different masses. Greenberg (2002) [3] noticed that such theory is nonlocal and acausal. The commutator of two fields is equal to the difference  $D^+(x - y; m) - D^-(x - y; \tilde{m})$  and does not vanish for space-like separation, unless the two masses are the same. In this sense the theory is not a Lorentz-invariant one. Moreover the Feynman propagator ( $T$ -product of fields) is

explicitly non-invariant. Indeed in momentum space it looks like

$$D_F(q) = \frac{1}{(2E(\mathbf{q}))} \frac{1}{(q_0 - E(\mathbf{q}))} - \frac{1}{(2\tilde{E}(\mathbf{q}))} \frac{1}{(q_0 + \tilde{E}(\mathbf{q}))} \quad (4)$$

and can be rewritten in invariant form only if  $m = \tilde{m}$ .

These arguments support a general “theorem” that any local fields theory that violates *CPT* symmetry necessarily violates Lorentz invariance. On the other hand, one can construct a nonlocal but Lorentz invariant theory which breaks CPT but conserves C, so the masses of particles and antiparticles remain equal [4].

Recently we have shown that theories with different masses for particle and antiparticle break some local conservation laws. In particular they break the electric current conservation [5]. Here we make a next step and argue that a non-zero mass difference between a charged particle and its antiparticle,  $\Delta m \neq 0$ , generates a non-zero mass of photon. In the theory where photon interacts with non-conserved electric current nothing protects photon from being massive and at the first loop one gets the non-zero  $m_\gamma$ :

$$m_\gamma^2 = C \frac{\alpha}{\pi} \Delta m^2. \quad (5)$$

The coefficient  $C$  can be calculated for any given convention about QFT with different masses for particle and antiparticle. In paper [5] we argued that there is no reasonable model for local QFT where  $m \neq \tilde{m}$ . Correspondingly there are no reliable theoretical frameworks for calculations of  $C$ . Still even with uncertain coefficient  $C$  the relation (5) is extremely interesting.

Indeed according to the PDG [1]:

$$|m_{e^+} - m_{e^-}|/m_{e^-} < 8 \cdot 10^{-9} \quad (6)$$

or  $\Delta m = |m_{e^+} - m_{e^-}| < 4 \cdot 10^{-3}$  eV.

Hence from relation (5) follows that the mass difference  $\Delta m$  generates the photon mass of the order:

$$m_\gamma^2 \sim C \left( \frac{\alpha}{\pi} \right) \Delta m^2 \leq 10^{-5} C \text{ eV}^2. \quad (7)$$

For any reasonable coefficient  $C$  and  $\Delta m$  which is not too far from the experimental upper bound (6) this value of  $m_\gamma$  is huge, much larger than the existing limits.

It is interesting to reformulate relation (7) in the opposite way, i.e. to say that an upper bound on the photon mass produces a bound on the mass difference for electron and positron. As it follows from eq. (7):

$$\Delta m_e < 20 m_\gamma / \sqrt{C}, \quad (8)$$

where we have to substitute for  $m_\gamma$  the upper limit on the photon mass. These limits and discussion of their validity are presented in the review [6].

The Earth based experiments give for the Compton wave length of the photon  $\lambda_C > 8 \cdot 10^7$  cm, i.e.  $m_\gamma < 3 \cdot 10^{-13}$  eV, and respectively  $\Delta m < 6 \cdot 10^{-12}$  eV, nine orders of magnitude stronger than (6).

From the measurement of the magnetic field of the Jupiter it follows that the Compton wave length of photon is larger than  $5 \cdot 10^{10}$  cm or  $m_\gamma < 4 \cdot 10^{-16}$  eV, and respectively  $\Delta m < 8 \cdot 10^{-15}$  eV.

The strongest solar system bound is obtained from the analysis of the solar wind extended up to the Pluto orbit [7]:  $\lambda_C > 2 \cdot 10^{13}$  cm, i.e.  $m_\gamma < 10^{-18}$  eV. This is an "official" limit present by the Particle Data Group [1]. The corresponding bound on the electron-positron mass difference is  $\Delta m < 2 \cdot 10^{-17}$  eV, which is almost 14 orders of magnitude stronger than the direct bound on  $\Delta m$ .

The strongest existing bound follows from the observation of the large scale magnetic fields in galaxies [8]:  $\lambda_C > 10^{22}$  cm and  $m_\gamma < 2 \cdot 10^{-27}$  eV. Correspondingly  $\Delta m < 4 \cdot 10^{-26}$  eV, which is 23 orders of magnitude stronger than the direct limit on the electron-positron mass difference.

The galactic bound on  $m_\gamma$  is subject to an uncertainty related to the way in which the electromagnetic  $U(1)$ -symmetry is broken. If this symmetry is broken in the soft Higgs-like way, the large scale magnetic fields may not be inhibited for massive photons due to formation of vortices where the symmetry is restored [9] and the bound presented above would be invalidated. However, in the case we consider here the photon mass surely does not originate from spontaneously broken gauge  $U(1)$  symmetry and the arguments of ref. [9] are not applicable.

Similar bounds can be derived on the CPT-odd mass differences of any other electrically charged particles and antiparticles.

It is instructive to present an example of actual calculations of  $m_\gamma$ . We stress again that there is no one sample of a local Lorentz invariant Field Theory with non-zero  $\Delta m$ . Therefore no calculation can be done in a formal self-consistent way. Here we simply start with 'a la Barenboim-Greenberg

decomposition for an electron-positron spinor field operator  $\Psi(x)$ :

$$\Psi(x) = \sum_{\mathbf{p}} \left\{ a(\mathbf{p}) \frac{u(p)e^{-ipx}}{\sqrt{2\omega(p)}} + b^+(\mathbf{p}) \frac{u(-\mathbf{p})e^{i\tilde{p}x}}{\sqrt{2\tilde{\omega}(p)}} \right\}, \quad (9)$$

$$\{a(\mathbf{p}), a^+(\mathbf{p}')\} = \delta_{\mathbf{p}, \mathbf{p}'}, \text{ etc.} \quad (10)$$

The first term in this decomposition annihilates electron with mass  $m$ , while the second term creates positron with mass  $\tilde{m}$ . Creation and annihilation operators obey the standard anti-commutation relations.

We also assume the validity of the usual local product of field operators for the electric current

$$j_\mu(x) = \bar{\Psi}(x)\gamma_\mu\Psi(x) \quad .$$

Because of the electron-positron mass difference this current is not conserved,  $\partial_\mu j(x) \neq 0$ .

The Feynman propagator (T-product of fields) is a sum of electron part contribution from the “Past” and of positron part contribution from the “Future”. The propagator is a covariant function only when  $m = \tilde{m}$ .

Consider the electron-positron pair contribution into the photon propagator, i.e. the polarization operator. Actually one can argue that polarization operator is still given by the textbook formula with covariant propagators with different masses  $m_1$  and  $m_2$  (we take  $m_1 = m$  and  $m_2 = \tilde{m}$ ):

$$\Pi_{\mu\nu} = (ie^2) \int \frac{d^D p}{(2\pi)^D} \text{Tr} \frac{1}{\hat{p} - m_1} \gamma_\nu \frac{1}{\hat{p} - \hat{q} - m_2} \gamma_\mu = \tilde{g}_{\mu\nu} \Pi_T(q^2) + g_{\mu\nu} \Pi_L(q^2), \quad (11)$$

where  $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$ . This divergent integral has to be regularized and to this end we choose the covariant dimensional regularization.

For the standard case  $m_1 = m_2$  the current is conserved and only the transverse part of polarization operator  $\Pi_T(q^2)$  is non-zero. Since there is no  $1/q^2$  singularity in the integral for  $\Pi_{\mu\nu}$ , the transverse polarization operator  $\Pi_T(q^2)$  has to vanish at  $q^2 = 0$ , i.e.  $\Pi_T(q^2) \sim q^2$  at  $q^2 \rightarrow 0$ . Thus  $\Pi_T$  gives no contribution into the photon mass.

For non-conserved currents a longitudinal function  $\Pi_L(q^2)$  would be generated. Nonzero  $\Pi_L(0) \neq 0$  corresponds to non-zero photon mass. Thus we are interested in the calculation of  $\Pi_L(q^2)$ . Consider the divergence of  $\Pi_{\mu\nu}$ ,

i.e.  $q_\mu \Pi_{\mu\nu}$ :

$$q_\mu \Pi_{\mu\nu} = q_\nu \Pi_L(q^2) = ie^2 \int \frac{d^D p}{(2\pi)^D} \text{Tr} \left[ \frac{1}{\hat{p} - m_1} \right] \gamma_\nu \left[ \frac{1}{\hat{p} - \hat{q} - m_L} \right] \hat{q} . \quad (12)$$

In the standard case of equal masses  $\hat{q}$  is equal to the difference of two inverse propagators and we reproduce the standard Ward identity. In our case  $\hat{q}$  is a difference of inverse propagators plus mass difference:

$$\hat{q} = (\hat{p} - m_1) - (\hat{p} - \hat{q} - m_2) + (m_1 - m_2) . \quad (13)$$

Substituting this formula into (12) we get a sum of 3 integrals. The first one is zero due to Lorentz covariance.

$$\int \frac{d^D p}{(2\pi)^D} \text{Tr} \gamma_\nu \frac{1}{\hat{p} - m_1} \equiv 0 . \quad (14)$$

The second one

$$\int \frac{d^D p}{(2\pi)^D} \text{Tr} \gamma_\nu \frac{1}{\hat{p} - \hat{q} - m_2} = 0 \quad (15)$$

is zero if regularization allows to make a shift of variables. The third integral is non-vanishing:

$$\begin{aligned} q_\nu \Pi_L(q^2) &= ie^2(m_1 - m_2) \int \frac{d^D p}{(2\pi)^D} \frac{\text{Tr} [(\hat{p} + m_1) \gamma_\nu (\hat{p} - \hat{q} + m_2)]}{(p^2 - m_1^2)[(p - q)^2 - m_2^2]} = \\ &= 4e^2(m_1 - m_2) q_\nu \int_0^1 dx \int \frac{d^D p}{(2\pi)^D} \frac{m_2 x - m_1(1 - x)}{[p^2 + \Delta^2]^2} , \end{aligned} \quad (16)$$

where  $\Delta^2 = m_1^2(1 - x) + m_2^2 x - q^2 x(1 - x)$ .

This integral is logarithmically divergent. At this moment we can forget about dimensional regularization.

For  $q^2 = 0$  this integral is trivial and one gets that

$$m_\gamma^2 = \Pi_L(0) = \frac{\alpha}{2\pi} [m_2 - m_1]^2 \left[ \ln \frac{\Lambda^2}{m^2} - \frac{5}{3} \right] , \quad (17)$$

where  $\Lambda$  is a cut-off, i.e. the photon mass is divergent and has to be renormalized. There is no principle that protect  $m_\gamma$  from being non-zero. Formally it can be an arbitrary number. But if loops have any physical sense for such

theories this number has to be proportional to the fine structure constant,  $\alpha$ , and disappears for equal mass, i.e. we arrive to eq. (5)

Similar arguments can be applied to the emergence of the nonzero graviton mass if the energy-momentum tensor is not conserved due to different masses of particles and antiparticles. A dimensional estimate for the graviton mass originating from the electron-positron loop with unequal masses of  $e^-$  and  $e^+$  is  $m_g^2 \sim \Delta m^2 \Lambda^2 / m_{Pl}^2$ , where  $\Lambda$  is the ultraviolet cut-off and  $m_{Pl}$  is the Planck mass. It is difficult to make any qualitative conclusion from this result but there is discontinuity between the zero mass limit of the theory with massive graviton and massless General Relativity [10]. This discontinuity is quite large and contradicts observations. A possible solution can be the Vainshtein mechanism [11] or one or other suggestion in modified gravity theories.

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